(64)分解因式及化簡

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(1)n^4+n^2+1

$$n^4+n^2+1=n^4+2n^2+1-n^2=(n^2+1)^2-n^2=(n^2+1+n)(n^2+1-n)$$
$$=(n^2+n+1)(n^2-n+1)$$

(2) $n^4+2n^3+n^2-1$

$$n^4+2n^3+n^2-1=(n^2+n)^2-1=(n^2+n+1)(n^2+n-1)$$

(3)$-n^4+n^2+2n+1$

$$-n^4+n^2+2n+1=-(n^4-n^2-2n-1)=-\left(n^4-(n^2+2n+1)\right)$$
$$=-(n^4-(n+1)^2)=-(n^2+n+1)(n^2-n-1)$$

(4)$\mathrm{n}^2+\frac{1}{n^2}-2$

$$\mathrm{n}^2+\frac{1}{n^2}-2=\frac{n^4+1-2n}{n^2}=\frac{n^4-2n^2+1}{n^2}=\frac{(n^2-1)^2}{n^2}$$

(5) $\frac{1}{n^2}-9$

$$\frac{1}{n^2}-9=(\frac{1}{n}+3)(\frac{1}{n}-3)$$

(6) $\frac{1}{n^2}+\frac{5}{n}+6$

$$\frac{1}{n^2}+\frac{5}{n}+6=\frac{1+5n+6n^2}{n^2}=\frac{6n^2+5n+1}{n^2}=\frac{(3n+1)(2n+1)}{n^2}$$
$$=\left(\frac{3n+1}{n}\right)\left(\frac{2n+1}{n}\right)=(\frac{1}{n}+3)(\frac{1}{n}+2)$$

(7)$\frac{2}{n}+n+3$

$$\frac{2}{n}+n+3=\frac{2+n^2+3n}{n}=\frac{n^2+3n+2}{n}=\frac{(n+1)(n+2)}{n}=\frac{(n+1)}{n}(n+2)$$
$$=\left(1+\frac{1}{n}\right)(n+2)=\left(\frac{1}{n}+1\right)(n+2)$$

化簡下列式子

(8)$\frac{n+1}{n+\frac{1}{n}+2}$

$$\frac{n+1}{n+\frac{1}{n}+2}=\frac{n+1}{\frac{n^2+2n+1}{n}}=\frac{(n+1)n}{(n+1)^2}=\frac{n}{n+1}$$

(9)$\frac{n+3}{(n+2)+\frac{1}{(n+2)}+2}$，$n\neq-3$

$$\frac{n+3}{(n+2)+\frac{1}{(n+2)}+2}=\frac{n+3}{\frac{(n+2)^2+2(n+2)+1}{n+2}}=\frac{(n+3)(n+2)}{(n+2+1)^2}$$

$$=\frac{(n+3)(n+2)}{(n+3)^2}=\frac{n+2}{n+3}$$

(10)$1+\frac{1}{1-\frac{1}{1+2n}}$

$$1+\frac{1}{1-\frac{1}{1+2n}}=1+\frac{1}{\frac{1+2n-1}{1+2n}}=1+\frac{1}{\frac{2n}{1+2n}}=1+\frac{1+2n}{2n}=\frac{2n+1+2n}{2n}$$

$$=\frac{4n+1}{2n}$$

(11)$\frac{\frac{3}{2n^2-n-1}}{\frac{2}{2n+1}-\frac{1}{n-1}}$

$$\frac{\frac{3}{2n^2-n-1}}{\frac{2}{2n+1}-\frac{1}{n-1}}=\frac{\frac{3}{(2n+1)(n-1)}}{\frac{2(n-1)-(2n+1)}{(2n+1)(n-1)}}=\frac{\frac{3}{(2n+1)(n-1)}}{\frac{2n-2-2n-1}{(2n+1)(n-1)}}=\frac{\frac{3}{(2n+1)(n-1)}}{\frac{-3}{(2n+1)(n-1)}}$$

$$=\frac{3}{(2n+1)(n-1)}\times\frac{(2n+1)(n-1)}{-3}=-1$$

(12)$\frac{\frac{1}{n^2-1}}{\frac{1}{n+1}-\frac{1}{n-1}}$

$$\frac{\frac{1}{n^2-1}}{\frac{1}{n+1}-\frac{1}{n-1}}=\frac{\frac{1}{(n+1)(n-1)}}{\frac{n-1-(n+1)}{(n+1)(n-1)}}=\frac{\frac{1}{(n+1)(n-1)}}{\frac{-2}{(n+1)(n-1)}}$$

$$=\frac{1}{(n+1)(n-1)}\times\frac{(n+1)(n-1)}{-2}=-\frac{1}{2}$$

(13) $\frac{\frac{1}{2n^2+3n+1}}{\frac{1}{n+1}-\frac{2}{2n+1}}$

$$\frac{\frac{1}{2n^2+3n+1}}{\frac{1}{n+1}-\frac{2}{2n+1}}=\frac{\frac{1}{(n+1)(2n+1)}}{\frac{2n+1-2(n+1)}{(n+1)(2n+1)}}=\frac{\frac{1}{(n+1)(2n+1)}}{\frac{-1}{(n+1)(2n+1)}}$$

$$=\frac{1}{(n+1)(2n+1)}\times\frac{(n+1)(2n+1)}{-1}=\frac{1}{-1}=-1$$