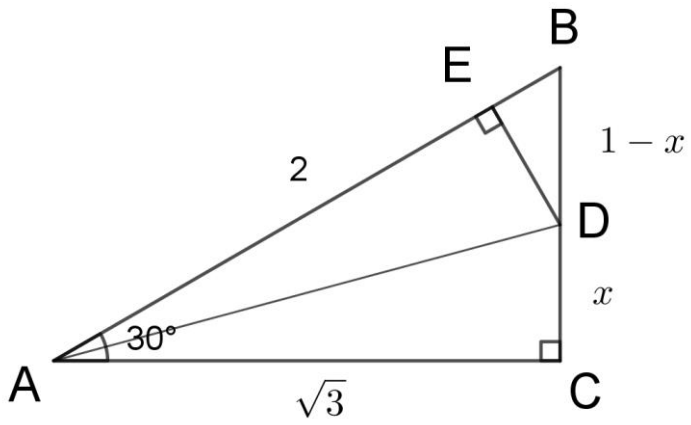
(14) 求$sin15°$



$\overline{AB}$為$\angle A$的分角線

因為$sin30° = \frac{1}{2}$，故令$\overline{BC} = 1$，$\overline{AB} = 2$，可得$\overline{AC} = \sqrt{3}$

令$\overline{DC} = x$，令$\overline{DE} \perp \overline{AB}$

$\because \Delta ADE \cong \Delta ADC \therefore \overline{DE} = x$

$\overline{BD} = 1 - x$

$\overline{BE} = 2 - \overline{AE} = 2 - \overline{AC} = 2 - \sqrt{3}$

$$sin\,B = sin\,60° = \frac{\sqrt{3}}{2} = \frac{\overline{DE}}{\overline{BD}} = \frac{x}{1-x}$$

$$2x = \sqrt{3} - \sqrt{3}x$$

$$x = \frac{\sqrt{3}}{2+\sqrt{3}} = \frac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \sqrt{3}(2-\sqrt{3})$$

$$x = 2\sqrt{3} - 3$$

$$\overline{AD}^2 = x^2 + (\sqrt{3})^2$$

$$= (2\sqrt{3} - 3)^2 + 3$$

$$= 24 - 12\sqrt{3} = 12(2 - \sqrt{3})$$

$\because sin15° = \dfrac{\overline{DC}}{\overline{AD}}$

$\therefore (sin15°)^2 = \dfrac{\overline{DC}^2}{\overline{AD}^2} = \dfrac{(2\sqrt{3}-3)^2}{12(2-\sqrt{3})}$

$= \dfrac{(2\sqrt{3}-3)^2\sqrt{3}}{12(2\sqrt{3}-3)}$

$= \dfrac{\sqrt{3}(2\sqrt{3}-3)}{12}$

$= \dfrac{6-3\sqrt{3}}{12}$

$= \dfrac{2-\sqrt{3}}{4} \cdots\cdots\cdots (1)$

$\therefore sin15° = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

令$2-\sqrt{3} = (x-y)^2$

$x^2 + y^2 - 2xy = 2 - \sqrt{3}$

$x^2 + y^2 = 2$

$-2xy = -\sqrt{3}$

得$x^2 + y^2 = 2$

$xy = \frac{\sqrt{3}}{2}$

$x = \frac{\sqrt{3}}{2y}$

得$\frac{3}{4y^2} + y^2 = 2$，同乘$4y^2$化簡得到$3 + 4y^4 = 8y^2$

也就是$4y^4 - 8y^2 + 3 = 0$

$(2y^2-3)(2y^2-1) = 0$

$\therefore y^2 = \frac{3}{2}$ 或 $y^2 = \frac{1}{2}$

$\therefore y = \frac{\sqrt{3}}{\sqrt{2}}$ 或 $y = \frac{1}{\sqrt{2}}$

可得$x = \frac{1}{\sqrt{2}}$ 或 $x = \frac{\sqrt{3}}{\sqrt{2}}$

可得$\sqrt{2-\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

代入(1)得$sin\,15° = \frac{\frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{2} = \frac{\frac{2\sqrt{3}}{\sqrt{2}} - \frac{2}{\sqrt{2}}}{2\times 2} = \frac{\sqrt{6}-\sqrt{2}}{4}$